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Rotational stiffnesses

1. INTRODUCTION

It is important to take into account the behaviour of the connections in the global analysis of the structure in order to obtain the correct distribution of internal forces, stresses and deformations.

To establish how the effect of the behaviour of the connections should be taken into account in the analysis of the structure, three different simplified models are used:

- Simple supports: those in which no moments are transmitted.

- Continuous: those in which the behaviour of the connections is not considered significantly in the analysis of the structure.

- Semi-continuous: those in which the behaviour of the connection must be taken into account in the global analysis of the structure.

As the program carries out an elastic analysis of the structure, the following ratios arise from the previous three models:

	Connection classification		
Type of model	Simple support	Continuous	Semi-continuous
Elastic analysis	Pinned connection	Rigid connection	Semi-rigid connection

The assumptions corresponding to semi-rigid connections are analysed below.

2. MOMENT-ROTATION DIAGRAM

The behaviour of the connections is studied by analysing the moment-rotation properties diagram, which allows to define the three main structural properties of a connection:

- Bending moment resistance $M_{j,Rd}$: Maximum ordinate of the diagram.

- Rotational stiffness S_j: the secant stiffness for an acting bending moment value $M_{j,Ed}$, defined up to rotation ϕ_{Xd} which corresponds to the point at which $M_{j,Ed}$ is equal to $M_{j,Rd}$.

- Rotation capacity ϕ_{Cd}: represents the maximum rotation of the moment-rotation diagram.

Figure 1 shows a typical connection moment-rotation diagram.

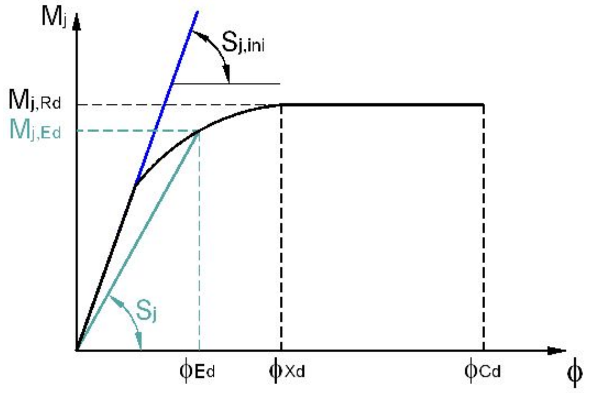

Figure 1

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According to its rotational stiffness S_j connections are classified as: pinned connections, rigid connections or semi-rigid connections. The boundaries between one type and another are shown in figure 2.

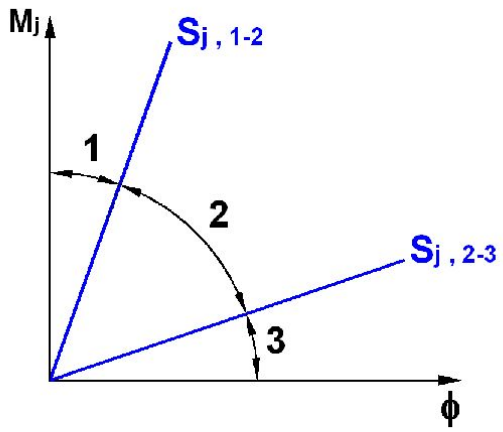

Figure 2

Where:

Zone 1 Rigid connections.
Zone 2 Semi-rigid connections.
Zone 3 Pinned connections.

3. CONNECTION ROTATIONAL STIFFNESS ANALYSIS

The program constructs the characteristic Myy moment - rotation in the xz plane for each connection at the end of an element where its rotational stiffness has been calculated due to the presence of deformable components in the node.

3.1. Calculation of initial stiffness Sj,ini

For axial forces not exceeding 5% of the transverse section's capacity, the stiffness S_j of the connection for the acting moment $M_{j,Ed}$ can be obtained using the following expression:

$$S_j = \frac{E \cdot z^2}{\mu g \sum_{i=1}^{i=n} \frac{1}{k_i}}$$

Where:

E Steel elastic modulus.
z Connection lever arm.
k_i Stiffness coefficient for the basic ith component.
μ Stiffness ratio:

$$\mu = \frac{S_{j,ini}}{S_j}$$

Coefficient μ can be established in the following way:

- For $M_{j,Ed} \leq 2/3 \cdot M_{j,Rd} \rightarrow \mu = 1.00$ (Elastic behaviour)

- For $2/3 \cdot M_{j,Rd} < M_{j,Ed} \leq M_{j,Rd}$:

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$$\mu = \left(1{,}5 \cdot \frac{M_{j,Ed}}{M_{j,Rd}}\right)^{2,7} \qquad (1.00 \leq \mu < 3.00)$$

To calculate the initial stiffness of connection $S_{j,ini}$ the component method is used, according to which:

- For column-beam connections:

$$S_{j,ini} = \frac{E \cdot z_{eq}^{2}}{\frac{1}{k_{eq}} + \frac{1}{k_1}}$$

- For splices and ridge joints:

$$S_{j,ini} = \frac{E \cdot z_{eq}^{2}}{\frac{1}{k_{eq}}}$$

Where:

z_{eq} Equivalent connection lever arm:

$$z_{eq} = \frac{\sum_{r=1}^{r=n} k_{eff,r} \cdot h_r^{2}}{\sum_{r=1}^{r=n} k_{eff,r} \cdot h_r}$$

h_r Distance between row r and the compression centre (which is considered to coincide with the compressed flange).

n Number of rows in tension.

k_1 Fixed joints of the beam and the column flange: Stiffness coefficient of the shear-resisting column web.
Fixed joints of the beam and the column web: Stiffness coefficient of the shear-resisting column flanges.

$$k_1 = \frac{0.38 \cdot A_{vc}}{\beta \cdot z_{eq}}$$

A_{vc} Shear area of the column panel.

β Force distribution coefficient, taken as $\beta = 1.0$.

k_{eq} Equivalent stiffness coefficient for the rows in tension of the connection:

$$k_{eq} = \frac{\sum_{r=1}^{r=n} k_{eff,r} \cdot h_r}{z_{eq}}$$

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$k_{eff,r}$ Effective stiffness coefficient of row r:

- For column-beam connections:

$$k_{eff,r} = \frac{1}{\frac{1}{k_3} + \frac{1}{k_4} + \frac{1}{k_5} + \frac{1}{k_{10}}}$$

- For splices and ridge joints:

$$k_{eff,r} = \frac{1}{\frac{1}{k_5} + \frac{1}{2 \cdot k_{10}}}$$

k_3 Fixed joints of the beam and the column flange: Stiffness coefficient of the tension-resisting column web.
Fixed joints of the beam and the column web: Stiffness coefficient of the tension-resisting vertical plate.

$$k_3 = \frac{0,7 \cdot l_{eff,1} \cdot t_{wc}}{d_c}$$

k_4 Fixed joints of the beam and the column flange: Stiffness coefficient of the flexure-resisting column flange.
Fixed joints of the beam and the column web: Stiffness coefficient of the flexure-resisting support plate.

$$k_4 = \frac{0,9 \cdot l_{eff,1} \cdot t_{fc}^{\ 3}}{m^3}$$

k_5 Stiffness coefficient of the moment-resisting end plate:

$$k_5 = \frac{0,9 \cdot l_{eff,1} \cdot t_p^{\ 3}}{m^3}$$

k_{10} Stiffness coefficient for the bolts in tension:

$$k_{10} = \frac{1,6 \cdot A_s}{L_b}$$

$l_{eff,1}$ Smallest value of the effective length of the bolt row, considered individually or in groups of rows.
m Distance from the bolt to the plastic hinge formed next to the section or next to the stiffener.
d_c Fixed joints of the beam and the column flange: Depth of the column web.
Fixed joints of the beam and the column web: Width of the vertical plate.
t_{wc} Fixed joints of the beam and the column flange: Thickness of the column web.
Fixed joints of the beam and the column web: Thickness of the vertical plate.
t_{fc} Fixed joints of the beam and the column flange: Thickness of the column flange.
Fixed joints of the beam and the column web: Thickness of the support plate.
t_p Thickness of the end plate.
A_s Tensile resistance area of the bolts.
L_b Bolt elongation length.

3.2. Calculation of plastic moment resistance Mj,Rd

The plastic moment resistance of connection $M_{j,Rd}$ is obtained as follows:

a) The minimum resistance of the zone in tension, for which the tensile resistance is obtained for each row of bolts, starting at the row furthest from the centre of compression, which supposedly coincides with the centre of the compressed flange to be joined. The smallest of the following values is taken for the resistance of each row of bolts:

- the tensile resistance of the column web (Fixed joints of the beam and the column flange).
- the tensile resistance of the vertical plate (Fixed joints of the beam and the column flange).
- the tensile resistance of the beam web.
- the flexural resistance of the column flange (Fixed joints of the beam and the column flange).
- the flexural resistance of the support plate (Fixed joints of the beam and the column web).
- the bending resistance of the end plate.
- the tensile resistance of the bolts.

b) The minimum resistance of the zone in compression is obtained, for which the smallest of the following values is taken:

- the compression resistance of the stiffeners (column-beam connections with stiffeners).
- the compression resistance of the column web (column-beam connections without stiffeners).
- the compression resistance of the section flange.

c) The shear resistance of the column panel (column-beam connections) is obtained.

d) The maximum admissible force of the group is taken as the smallest value amongst:

- the minimum resistance of the zone in tension.
- the minimum resistance of the zone in compression.
- the shear resistance of the panel of the column.

e) The maximum force is shared amongst the rows in tension, starting at the row furthest away from the centre of compression in such a way that the force assigned to each row $F_{TEd,i}$ does not exceed the previously calculated capacity.

f) The moment resistance of connection Mj,Rd is established using the following expression:

$$M_{j,Rd} = \sum_{i=1}^{n} h_i \cdot F_{T,Ed,i}$$

Where:

h_i Distance between the bolt row with index 'i' and the centre of compression.

$F_{TEd,i}$ Tensile force in the ith row.

n Number of bolt rows situated in the tension zone of the connection.

3.3. Moment-rotation diagram representation

The moment rotation diagram of the connection is drawn using the initial stiffness $S_{j,ini}$ and the moment resistance $M_{j,Rd}$, for positive and negative values and the forces acting at the ends of the element. The result (curve A) is shown in Figure 3:

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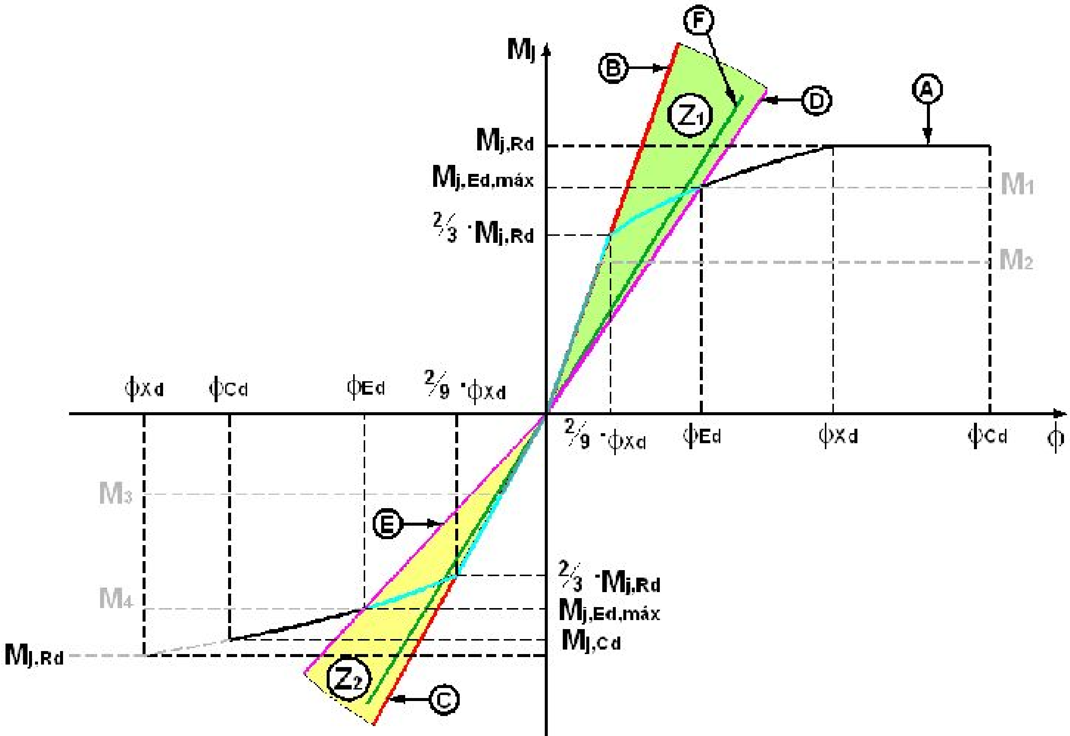

Figure 3

The following data can be seen in the graph:

a) Characteristic points of diagram 'A':

- $M_{j,Rd}$ Plastic moment resistance of the connection.
- $2/3 \cdot M_{j,Rd}$ Elastic moment resistance of the connection.
- $M_{j,Cd}$ Moment resistance corresponding to the rotation capacity of the connection ϕ_{Cd}.
- $M_{j,Edmax}$ Maximum moment of all the combinations (with or without earthquake loading).
- ϕ_{Xd} Rotation corresponding to the plastic moment resistance.
- $2/9 \cdot \phi_{Xd}$ Rotation corresponding to the elastic moment resistance.
- ϕ_{Cd} Rotation capacity of the connection.

When rotation ϕ_{Cd} is less than ϕ_{Xd}, diagram 'A' is interrupted at the corresponding point ($M_{j,Cd}$; ϕ_{Cd}), as the connection cannot reach rotation ϕ_{Xd} for moment resistance $M_{j,Rd}$.

b) Lines:

- B Line whose gradient is the rotational stiffness corresponding to the smallest acting positive moment.
- C Line whose gradient is the rotational stiffness corresponding to the smallest acting negative moment.
- D Line whose gradient is the rotational stiffness corresponding to the greatest acting positive moment.
- E Line whose gradient is the rotational stiffness corresponding to the greatest acting negative moment.
- F Line whose gradient is the rotational stiffness used in the analysis of the structure.

Also displayed are the lines whose gradients are the initial rotational stiffness for positive moments as well as for negative moments.

c) Zones:

- Z_1 Zone which includes the rotational stiffness values for the range of positive moments.
- Z_2 Zone which includes the rotational stiffness values for the range of negative moments.

3.4. Application of the moment-rotation diagram

3.4.1. Rotational stiffness in plane xz of the connection

The program proposes to adopt a rotational stiffness value equal to the smallest of those corresponding to lines D and E of Figure 3.

When, at an element end, there is a significant interaction of Myy moment with the axial forces and/or Mzz bending moments (xy plane of the connection), this type of stiffness analysis loses precision and the program warns of this situation if the following condition is not met:

$$\frac{M_{z,Ed}}{M_{z,pl,Rd}} + \frac{N_{Ed}}{N_{pl,Rd}} \leq 0.05$$

Where:

$M_{z,Ed}$ Moment Mzz.

$M_{z,pl,Ed}$ Plastic moment resistance of the section.

N_{Ed} Axial force.

$N_{pl,Rd}$ Plastic axial force resistance of the section.

3.4.2. Rotational stiffness in plane xy of the connection

Bearing in mind that the Mzz moments acting on a connection are usually negligible compared to the Myy moments, the program proposes a rotational stiffness value corresponding to the initial rotational stiffness $S_{j,ini}$ of the xy plane, calculated in a similar way as for the xz plane.

4. CRITERIA FOR CONNECTION DESIGN

Bolted connections designed by the program always have a critical failure mode conditioned by the bending of the column flange or end plate, which allows for a plastic distribution of the tensile forces on the bolts.

The program avoids connections with insufficient rotation capacity, applying an optimum ratio between the thickness of the plate and the diameter of the bolt.

If flexible plates are applied, leverage effects are developed which cause the tensile forces to increase in the bolts. The program takes these effects into account in the checks it undergoes on bolts in tension, with or without shear interaction.

The program excludes the possibility of weld fracture by designing them with a resistance equal to or greater than the section joined to the end plate.

The designed connections possess the required resistance to support the forces resulting from the global analysis of the structure.

The conditions established by the program to group nodes and apply the same connections are as follows:

- The spatial layout of the bars converging at the node must be the same.

- Similar steel bars must have the same section and be of the same steel class.

- Homologous element ends should have the same type of fixity. If rotational stiffness values have been defined, their difference must lie within a 10% boundary.